A Synopsis of the Minimal Modal Interpretation of Quantum Theory

Jacob A. Barandes[1, *] and David Kagan[2, †]
[1] *Jefferson Physical Laboratory, Harvard University, Cambridge, MA 02138*
[2] *Department of Physics, University of Massachusetts Dartmouth, North Dartmouth, MA 02747*

(Dated: May 5, 2017)

We summarize a new realist, unextravagant interpretation of quantum theory that builds on the existing physical structure of the theory and allows experiments to have definite outcomes but leaves the theory's basic dynamical content essentially intact. Much as classical systems have specific states that evolve along definite trajectories through configuration spaces, the traditional formulation of quantum theory permits assuming that closed quantum systems have specific states that evolve unitarily along definite trajectories through Hilbert spaces, and our interpretation extends this intuitive picture of states and Hilbert-space trajectories to the more realistic case of open quantum systems despite the generic development of entanglement. Our interpretation—which we claim is ultimately compatible with Lorentz invariance—reformulates wave-function collapse in terms of an underlying interpolating dynamics, makes it possible to derive the Born rule from deeper principles, and resolves several open questions regarding ontological stability and dynamics.

I. INTRODUCTION

In this letter and in a more comprehensive companion paper [1], we present a realist interpretation of quantum theory that hews closely to the basic structure of the theory in its widely accepted current form. Our primary goal is to move beyond instrumentalism and describe an actual reality that lies behind the mathematical formalism of quantum theory. We also intend to provide new hope to those who find themselves disappointed with the pace of progress on making sense of the theory's foundations [2, 3].

A. Why Do We Need a New Interpretation?

Despite the absence of a clear consensus on all the Copenhagen interpretation's precise metaphysical commitments, it is still, at least according to some surveys [4, 5], the most popular interpretation of quantum theory today, but it also suffers from a number of serious drawbacks. Most significantly, the definition of a measurement according to the Copenhagen interpretation relies on a physically questionable demarcation, known as the Heisenberg cut (*Heisenbergscher Schnitt*) [6, 7], between the large classical systems that carry out measurements and the small quantum systems that they measure. (See Figure 1.) This ill-defined Heisenberg cut has never been identified in any experiment to date and must be put into the interpretation by hand. An associated issue is the interpretation's assumption of wave-function collapse, by which we refer to the supposedly instantaneous, discontinuous change in a quantum system immediately following a measurement by a classical system, in stark contrast to the smooth time evolution that governs dynamically closed systems.

The Copenhagen interpretation is also unclear as to the ultimate meaning of the state vector of a system: Does a system's state vector merely represent the experimenter's knowledge, is it some sort of objective probability distribution,[1] or is it an irreducible ingredient of reality like the state of a classical system [11]? For that matter, what constitutes an observer, and can we meaningfully talk about the state of an observer within the formalism of quantum theory? Given that no realistic system is ever perfectly free of quantum entanglement with other systems, and thus no realistic system can ever truly be assigned a specific state vector in the first place, what becomes of the popular depiction of quantum theory in which every particle is purportedly described by a specific wave function propagating in three-dimensional space? The Copenhagen interpretation leads to additional trouble when trying to make sense of thought experiments like Schrödinger's cat, Wigner's friend, and the quantum Zeno paradox.[2]

A more satisfactory interpretation would eliminate the need for an *ad hoc* Heisenberg cut, thereby demoting measurements to an ordinary kind of interaction and allowing quantum theory to be a complete theory that seamlessly encompasses *all* systems in Nature, including observers as physical systems with quantum states of their own. Moreover, an acceptable interpretation

* barandes@physics.harvard.edu
† dkagan@umassd.edu

[1] Recent work [8–10] casts considerable doubt on assertions that state vectors are nothing more than probability distributions over more fundamental ingredients of reality.

[2] We discuss all these thought experiments in our companion paper [1].

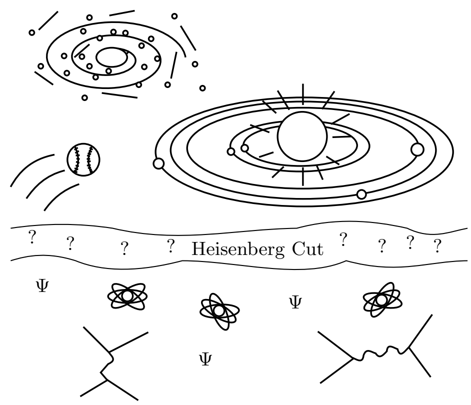

Figure 1. The Heisenberg cut.

should *fundamentally* (even if not always *superficially*) be consistent with all experimental data and other reliably known features of Nature, including relativity, and should be general enough to accommodate the large variety of both presently known and hypothetical physical systems. Such an interpretation should also address the key no-go theorems developed over the years by researchers working on the foundations of quantum theory; should not depend on concepts or quantities whose definitions require a physically unrealistic measure-zero level of sharpness; and should be insensitive to potentially unknowable features of reality, such as whether we can sensibly define "the universe as a whole" as being a closed or open system.

B. Motivation from Classical Physics

Our interpretation is fully quantum in nature. However, for purposes of motivation, consider the basic theoretical structure of classical physics: A classical system has a specific state—which we call its "ontic state"—that evolves in time through the system's configuration space according to some dynamical rule that may or may not be stochastic, and this dynamical rule exists whether or not the system's state lies beneath a nontrivial evolving probability distribution—called an "epistemic state"—on the system's configuration space; moreover, the dynamical rule for the system's underlying ontic state is consistent with the overall evolution of the system's epistemic state, in the sense that if we consider a probabilistic ensemble over the system's initial ontic state and apply the dynamical rule to each ontic state in the ensemble, then we correctly obtain the overall evolution of the system's epistemic state as a whole.

In particular, note the *insufficiency* of specifying a dynamical rule *solely* for the evolution of the system's overall probability distribution—that is, for its epistemic state alone—but *not* for the system's underlying ontic state itself, because then the system's underlying ontic state would be free to fluctuate wildly and discontinuously between macroscopically distinct configurations. For example, even in the simple case in which a classical system's epistemic state describes constant probabilities p_1 and p_2 for the system to be in respective macroscopically distinct ontic states q_1 or q_2, there would be nothing preventing the system's ontic state from hopping discontinuously between q_1 and q_2 with respective frequency ratios p_1 and p_2 over arbitrarily short time intervals. Essentially, by imposing a dynamical rule on the system's underlying ontic state, we can provide a "smoothness condition" for the system's hidden physical configuration over time and thus eliminate these kinds of instabilities. ("Hidden variables need hidden dynamics.")

In quantum theory, a system that is *exactly* closed and that is *exactly* in a pure state (both conditions that are unphysical idealizations) evolves along a well-defined trajectory through the system's Hilbert space according to a well-known dynamical rule, namely, the Schrödinger equation. However, in traditional formulations of quantum theory, an *open* quantum system that must be described by a density matrix due to entanglement with other systems—that is, a system in a so-called improper mixture—does not have a specific underlying ontic state vector, let alone a Hilbert-space trajectory or a dynamical rule governing the time evolution of such an underlying ontic state vector and consistent with the overall evolution of the system's density matrix.

It is a chief goal of our interpretation of quantum theory to provide these missing ingredients—in large part by assigning an explicit meaning to improper mixtures. In a sense that we will make much more precise, our interpretation of quantum theory asserts that systems have actual states that evolve along kinematical trajectories through their state spaces, and that those trajectories are governed by specific (if approximate) dynamical rules.

II. THE MINIMAL MODAL INTERPRETATION

A. Conceptual Summary

In short, rather than invoking the Born rule together with a collapse postulate that converts *improper* mixtures into *proper* mixtures—that is, into classical probability distributions over sets of definite outcomes—we instead attach an interpretation directly to improper mixtures: For a quantum system in a fully improper mixture, our new interpretation identifies the *eigenstates* of the system's density matrix with the *possible states* of the system in reality and identifies the *eigenvalues* of that density matrix with the *probabilities* that one of those possible states is *actually* occupied. Our interpretation introduces just enough minimal structure beyond that simple picture to provide a dynamical rule for underlying state vectors as they evolve along Hilbert-space trajectories and to evade criticisms made in the past regarding similar interpretations. This minimal additional structure consists of a new class of conditional probabilities amounting essentially to a series of smoothness conditions that kinematically relate the states of parent systems to the states of their subsystems, as well as dynamically relate the states of a single system to each other over time.

B. Technical Summary

Our new interpretation, which we call the *minimal modal interpretation* of quantum theory and which we motivate and detail more extensively in our companion paper [1], consists of several parsimonious ingredients:

1. **Ontic States:** We define quantum ontic states Ψ_i—meaning the states of a given system as it could actually exist in reality—in terms of arbitrary (unit-norm) state vectors $|\Psi_i\rangle$ in the system's Hilbert space $\mathcal{H}$:
$$\Psi_i \leftrightarrow |\Psi_i\rangle \in \mathcal{H} \;\; \text{(up to overall phase)}. \tag{1}$$
This definition is the quantum counterpart to the classical notion of ontic states as elements in a configuration space.

2. **Epistemic States:** We define quantum epistemic states $\{(p_i, \Psi_i)\}_i$ as probability distributions over sets of possible quantum ontic states,
$$\{(p_i, \Psi_i)\}_i, \qquad p_i \in [0,1],$$
where, again, this definition parallels the corresponding notion from classical physics. We translate logical mutual exclusivity of ontic states Ψ_i as mutual orthogonality of state vectors $|\Psi_i\rangle$, and we make a distinction between subjective epistemic states (proper mixtures) and objective epistemic states (improper mixtures): The former arise from merely classical ignorance and are uncontroversial, whereas the latter arise from quantum entanglement with other systems and do not have a widely accepted *a priori* meaning outside of our interpretation of quantum theory. Indeed, the problem of interpreting objective epistemic states may well be unavoidable: All realistic systems are entangled with other systems to a nonzero degree and thus cannot be described exactly by pure states or by purely subjective epistemic states.[3] As part of our introduction of epistemic states into quantum theory, we posit a correspondence between objective epistemic states $\{(p_i, \Psi_i)\}_i$ and density matrices $\hat{\rho}$:
$$\{(p_i, \Psi_i)\}_i \leftrightarrow \hat{\rho} = \sum_i p_i \, |\Psi_i\rangle \langle \Psi_i| \,. \tag{2}$$
(See Figure 2.) The relationship between subjective epistemic states and density matrices is not as strict, as we explain in our companion paper [1].

3. **Partial Traces:** We invoke the partial-trace operation $\hat{\rho}_Q \equiv \mathrm{Tr}_E[\hat{\rho}_{Q+E}]$, motivated and defined in our companion paper [1] without appeals to the Born rule or Born-rule-based averages, to relate the density matrix (and thus the epistemic state) of any subsystem Q to that of any parent system $W = Q + E$.

4. **Quantum Conditional Probabilities:** We introduce a general class of quantum conditional probabilities,
$$\begin{aligned} &p_{Q_1,\ldots,Q_n|W}\left(i_1,\ldots,i_n;t'|w;t\right) \\ &\equiv \mathrm{Tr}_W\left[\left(\hat{P}_{Q_1}\left(i_1;t'\right)\otimes\cdots\otimes\hat{P}_{Q_n}\left(i_n;t'\right)\right)\mathcal{E}_W^{t'\leftarrow t}[\hat{P}_W\left(w;t\right)]\right] \\ &\sim \mathrm{Tr}[\hat{P}_{i_1}\left(t'\right)\cdots\hat{P}_{i_n}\left(t'\right)\mathcal{E}[\hat{P}_w\left(t\right)]], \end{aligned} \tag{3}$$
relating the possible ontic states of any partitioning collection of mutually disjoint subsystems $Q_1,\ldots,Q_n$ to the possible ontic states of a corresponding parent system $W = Q_1 + \cdots + Q_n$ whose own dynamics is governed approximately by a linear completely-positive-trace-preserving ("CPTP") dynamical mapping $\mathcal{E}_W^{t'\leftarrow t}$ over the given time interval $t' - t$. Here the operator $\hat{P}_W(w;t)$ denotes the eigenprojector onto the eigenstate $|\Psi_W(w;t)\rangle$ of the density matrix $\hat{\rho}_W(t)$ of the parent system W at the initial time t, and, similarly, for $\alpha = 1,\ldots,n$, the operator $\hat{P}_{Q_\alpha}(i;t')$ denotes the eigenprojector onto the eigenstate $|\Psi_{Q_\alpha}(i;t')\rangle$ of the density matrix $\hat{\rho}_{Q_\alpha}(t')$ of the subsystem Q_α at the final time t'. In a rough sense, the dynamical mapping $\mathcal{E}_W^{t'\leftarrow t}$ acts as a parallel-transport superoperator that moves the parent-system eigenprojector $\hat{P}_W(w;t)$ from t to t' before we compare it with the subsystem eigenprojectors $\hat{P}_{Q_\alpha}(i;t')$.

In generalizing unitary dynamics to linear CPTP dynamics in this manner, as is necessary in order to account for the crucial and non-reductive quantum relationships between parent systems and their subsystems, note that we are *not* proposing any fundamental modifications to the dynamics of quantum theory nor any new sources of violations of time-reversal symmetry, such as occur in GRW-type spontaneous-localization models [12], but

[3] As we explain in Section II E, there are reasons to be skeptical of the common assumption that one can always assign an exactly pure state or purely subjective epistemic state to "the universe as a whole."

are simply accommodating the fact that generic mesoscopic and macroscopic quantum systems are typically open to their environments to some nonzero degree. Indeed, linear CPTP dynamical mappings are widely used in quantum chemistry as well as in quantum information science, in which they are known as quantum operations; when specifically regarded as carriers of quantum information, they are usually called quantum channels.[4] A well-known, concrete example is the Lindblad equation [21].

As one special case, our quantum conditional probabilities provide a *kinematical* smoothing relationship

$$\begin{aligned} &p_{Q_1,\dots,Q_n|W}\left(i_1,\dots,i_n|w\right)\\ &\equiv \mathrm{Tr}_W\left[\left(\hat{P}_{Q_1}\left(i_1\right)\otimes\cdots\otimes\hat{P}_{Q_n}\left(i_n\right)\right)\hat{P}_W\left(w\right)\right]\\ &=\left\langle\Psi_{W,w}\right|\left(\left|\Psi_{Q_1,i_1}\right\rangle\left\langle\Psi_{Q_1,i_1}\right|\otimes\cdots\right.\\ &\qquad\left.\otimes\left|\Psi_{Q_n,i_n}\right\rangle\left\langle\Psi_{Q_n,i_n}\right|\right)\left|\Psi_{W,w}\right\rangle \end{aligned} \tag{4}$$

between the possible ontic states of any partitioning collection of mutually disjoint subsystems $Q_1,\dots,Q_n$ and the possible ontic states of the corresponding parent system $W = Q_1+\cdots+Q_n$ at any *single* moment in time. As another special case, if we take $Q\equiv Q_1 = W$, then our quantum conditional probabilities also provide a *dynamical* smoothing relationship

$$\begin{aligned} p_Q\left(i';t'|i;t\right) &\equiv \mathrm{Tr}_Q[\hat{P}_Q\left(i';t'\right)\mathcal{E}_Q^{t'\leftarrow t}[\hat{P}_Q\left(i;t\right)]]\\ &\sim \mathrm{Tr}[\hat{P}_{i'}\left(t'\right)\mathcal{E}[\hat{P}_i\left(t\right)]] \end{aligned} \tag{5}$$

between the possible ontic states of the system Q over time and also between the objective epistemic states of the system Q over time.

Essentially, 1 establishes a linkage between ontic states and elements of Hilbert spaces, 2 establishes a linkage between (objective) epistemic states and density matrices, 3 establishes a linkage between parent-system density matrices and subsystem density matrices, and 4 establishes a linkage between parent-system ontic states and subsystem ontic states as well as between parent-system epistemic states and subsystem epistemic states, either at the same time or at different times.

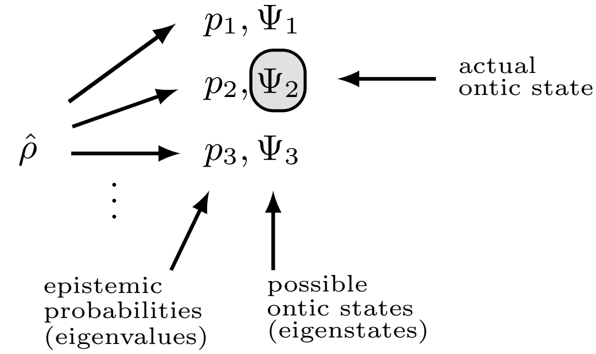

Figure 2. A schematic depiction of our postulated relationship between a system's density matrix $\hat{\rho}$ and its associated epistemic state $\{(p_1,\Psi_1),(p_2,\Psi_2),(p_3,\Psi_3),\dots\}$. The latter consists of epistemic probabilities $p_1,p_2,p_3,\dots$ (the eigenvalues of the density matrix) and possible ontic states $\Psi_1,\Psi_2,\Psi_3,\dots$ (represented by the eigenstates of the density matrix), where one of those *possible* ontic states (in this example, Ψ_2) is the system's *actual* ontic state.

After verifying in our companion paper [1] that our quantum conditional probabilities satisfy a number of consistency requirements, we show that they allow us to avoid ontological instabilities that have presented problems for other modal interpretations, analyze the measurement process, study various familiar "paradoxes" and thought experiments, and examine the status of Lorentz invariance and locality in our interpretation of quantum theory. In particular, our interpretation accommodates the nonlocality implied by the EPR-Bohm and GHZ-Mermin thought experiments without leading to superluminal signaling, and evades claims by Myrvold [22] purporting to show that interpretations like our own lead to unacceptable ontological contradictions with Lorentz invariance. As a consequence of its compatibility with Lorentz invariance, we claim that our interpretation is capable of encompassing all the familiar quantum models of physical systems in widespread use today, from nonrelativistic point particles to quantum field theories and even string theory.

C. Modal Interpretations

Our use of the modifiers "possible and "actual," together known formally as modalities, identifies our interpretation of quantum theory as belonging to the general class of modal interpretations originally introduced by Krips in 1969 [23–25] and then independently developed by van Fraassen (whose early formulations involved the fusion of modal logic with quantum logic), Dieks, Vermaas, and others [26–30].

---

[4] See [13–15] for early work in this direction, and see [16] for a modern pedagogical review. Starting from a simple measure of distinguishability between density matrices that is non-increasing under linear CPTP dynamics [17], one can argue [18, 19] that linear CPTP dynamics implies the absence of any backward flow of information into the system from its environment. [20] strengthens this reasoning by proving that exact linear CPTP dynamics exists for a given quantum system if and only if the system's initial correlations with its environment satisfy a quantum data-processing inequality that prevents backward information flow.

The modal interpretations are now understood to encompass a very large set of interpretations of quantum theory, including most interpretations that fall between the "many worlds" of the Everett-DeWitt approach and the "no worlds" of the instrumentalist approaches. Generally speaking, in a modal interpretation, one singles out some preferred basis for each system's Hilbert space and then regards the elements of that basis as being the system's *possible* ontic states—one of which is the system's *actual* ontic state—much in keeping with how we think conceptually about classical probability distributions. For example, as emphasized in [30], the traditional de Broglie-Bohm pilot-wave interpretation [31–33] can be regarded as being a special kind of modal interpretation in which the preferred basis is permanently fixed *for all* systems at a universal choice. Other modal interpretations, such as our own, instead allow the preferred basis for a given system to change—in our case by choosing the preferred basis to be the evolving eigenbasis of that system's density matrix. However, we claim that no existing modal interpretation captures the one that we summarize in this letter and describe more fully in our companion paper [1].

D. Hidden Variables and the Irreducibility of Ontic States

To the extent that our interpretation of quantum theory involves hidden variables, the actual ontic states underlying the epistemic states of systems play that role. However, one could also argue that calling them hidden variables is just a matter of semantics because they are on the same metaphysical footing as both the *traditional* notion of quantum states as well as the actual ontic states of *classical* systems.

In any event, it is important to note that our interpretation includes *no other* hidden variables: Just as in the classical case, we regard ontic states as being *irreducible* objects, and, in keeping with this interpretation, we *do not* regard a system's ontic state itself as being an epistemic probability distribution—much less a "pilot wave"—over a set of more basic hidden variables. In a rough sense, our interpretation *unifies* the de Broglie-Bohm interpretation's pilot wave and hidden variables into a single ontological entity that we call an ontic state.

In particular, we do not attach an epistemic probability interpretation to the *components* of a vector representing a system's ontic state, nor do we assume *a priori* the Born rule, which we ultimately *derive* in our companion paper [1] as a means of computing empirical outcome probabilities. Otherwise, we would need to introduce an unnecessary *additional* level of probabilities into our interpretation and thereby reduce its axiomatic parsimony and explanatory power.

Via the phenomenon of environmental decoherence, our interpretation ensures that the evolving ontic state of a sufficiently macroscopic system—with significant energy and in contact with a larger environment—is highly likely to be represented by a temporal sequence of state vectors whose labels evolve in time according to recognizable semiclassical equations of motion. For microscopic, isolated systems, by contrast, we simply accept that the ontic state vector may not usually have an intuitively familiar classical description.

E. Comparison with Other Interpretations of Quantum Theory

Our interpretation, which builds on the work of many others, is general, model-independent, and encompasses relativistic systems, but is also conservative and unextravagant: It includes only metaphysical objects that are either already a standard part of quantum theory or that have counterparts in classical physics. We do not posit the existence of exotic "many worlds" [34–37], physical "pilot waves" [31–33], or any fundamental GRW-type dynamical-collapse or spontaneous-localization modifications to quantum theory [12, 38–40]. Indeed, our interpretation leaves the widely accepted mathematical structure of quantum theory essentially intact.[5] At the same time, we argue in our companion paper [1] that our interpretation is ultimately compatible with Lorentz invariance and is nonlocal only in the mild sense familiar from the framework of classical gauge theories.

Furthermore, we make no assumptions about as-yet-unknown aspects of reality, such as the fundamental discreteness or continuity of time or the dimensionality of the ultimate Hilbert space of Nature. Nor does our interpretation rely in any crucial way upon the existence of a well-defined maximal parent system that encompasses all other systems and is dynamically closed in the sense of having a so-called cosmic pure state or universal wave function that *precisely* obeys the Schrödinger equation; by contrast, this sort of unsubstantiated cosmic assumption is a necessarily *exact* ingredient in the traditional

[5] Moreover, our interpretation does not introduce any new violations of time-reversal symmetry and gives no fundamental role to relative states [34]; a cosmic multiverse or self-locating uncertainty [41]; coarse-grained histories or decoherence functionals [42, 43]; decision theory [44, 45]; Dutch-book coherence, SIC-POVMs, or *urgleichungs* [46]; circular frequentist arguments involving unphysical "limits" of infinitely many copies of measurement devices [41, 47, 48]; infinite imaginary ensembles [49]; quantum reference systems or perspectivalism [50, 51]; relational or non-global quantum states [52–55]; many-minds states [56]; mirror states [54, 55]; faux-Boolean algebras [30, 57]; "atomic" subsystems [58]; algebraic quantum field theory [59]; secret classical superdeterminism or fundamental information loss [60]; cellular automata [61]; classical matrix degrees of freedom or trace dynamics [62]; or discrete Hilbert spaces or appeals to unknown Planck-scale physics [48].

formulations of the de Broglie-Bohm pilot-wave interpretation and the Everett-DeWitt many-worlds interpretation.

Indeed, by considering merely the *possibility* that our observable universe is but a small region of an eternally inflating cosmos of indeterminate spatial size and age [63–65], it becomes clear that the idea of a biggest closed system containing all other systems ("the universe as a whole") may not generally be a sensible or empirically verifiable concept to begin with, let alone an axiom on which a robust interpretation of quantum theory can safely rely. Our interpretation certainly allows for the existence of a biggest closed system in an objectively pure state, but is also fully able to accommodate the alternative circumstance that if we were to imagine gradually enlarging our scope to parent systems of increasing physical size, then we might well find that the hierarchical sequence never terminates at any maximal, dynamically closed system, but may instead lead to an unending "Russian-doll" succession of ever-more-open parent systems in objectively mixed states representing improper mixtures.[6]

III. MEASUREMENTS AND LORENTZ INVARIANCE

A. Von Neumann Measurements and the Born Rule

For the purposes of establishing how our minimal modal interpretation makes sense of measurements, how decoherence turns the environment into a "many-dimensional chisel" that rapidly sculpts the ontic states of systems into their precise shapes,[7] and how the Born rule naturally emerges to an excellent approximation, we consider in our companion paper [1] the idealized example of a so-called Von Neumann measurement. Along the way, we also address the status of both the measurement problem generally and the notion of wave-function collapse specifically in the context of our interpretation of quantum theory. Ultimately, we find that our interpretation solves the measurement problem by replacing instantaneous axiomatic wave-function collapse with an interpolating ontic-level dynamics, and thereby eliminates the need for an *ad hoc* Heisenberg cut.

B. The Myrvold No-Go Theorem

Building on a paper by Dickson and Clifton [66] and employing arguments similar to Hardy [67], Myrvold [22] argued that modal interpretations are fundamentally inconsistent with Lorentz invariance at a deeper level than mere unobservable nonlocality, leading to much additional work [53, 59, 68] in subsequent years to determine the implications of his result. Specifically, Myrvold argued that assuming the existence of ontic states underlying density matrices could lead to paradoxes arising from Lorentz transformations. Seemingly the only way to avoid this conclusion would then be to break Lorentz symmetry in a fundamentally ontological way by asserting the existence of a universal "preferred" reference frame in which all ontic-state assignments must be made.

Myrvold's claims, and those of Dickson and Clifton as well as Hardy, rest on assumptions that do not hold in our interpretation of quantum theory. In particular, Dickson and Clifton [66] assume that hidden ontic states admit certain joint epistemic probabilities that are conditioned on *multiple* disjoint systems at an initial time, and such probabilities are not a part of our interpretation, as we explain in our companion paper [1].[8] Similarly, all of Myrvold's arguments hinge on the assumed existence of joint epistemic probabilities for two disjoint systems at *two or more separate times*—some of which are, moreover, *Lorentz-transformed* times—and again such probabilities are not present in our interpretation. As Dickson and Clifton point out in Appendix B of their paper, Hardy's argument also depends on several inadmissible assumptions about ontic property assignments in modal interpretations.

C. Quantum Theory and Classical Gauge Theories

Suppose that we were to imagine reifying all the *possible* ontic states defined by every system's density matrix as simultaneous *actual* ontic states in the sense of

[6] To avoid this problem, one might try to argue that one can always *formally* define a closed maximal parent system—presumably in an objectively pure state—just to be "the system containing all systems." Whatever logicians might say about such a construction, we run into the more prosaic issue that if we cannot construct this closed maximal parent system via a well-defined succession of parent systems of incrementally increasing size, then it becomes unclear mathematically how we can generally define any human-scale system as a subsystem of the maximal parent system and thereby define the partial-trace operation. Furthermore, if our observable cosmic region is indeed an open system, then its own time evolution may not be exactly linear—an important feature of generic open systems that is rarely acknowledged in the literature—in which case it is far from obvious that we can safely and rigorously embed that open-system time evolution into the hypothetical unitary dynamics of any conceivable closed parent system.

[7] The way that decoherence sculpts ontic states into shape is reminiscent of the way that the external pressure of air molecules above a basin of water maintains the water in its liquid phase.

[8] The same implicit assumption occurs in Section 9.2 of [30].

the many-worlds interpretation.[9] Then because every density matrix *as a whole* evolves locally and there is no single actual ontic state that jumps between different possible ontic states, no nonlocal dynamics between the actual ontic states is necessary and our interpretation of quantum theory becomes manifestly dynamically local: For example, each spin detector in the EPR-Bohm or GHZ-Mermin thought experiments possesses all its possible results in actuality, and the larger measurement that later compares the results of the spin detectors apparatus locally "splits" into all the various possibilities when it visits each spin detector and looks at the detector's final reading.

In our companion paper [1], we argue that we can make sense of this step of adding unphysical actual ontic states into our interpretation of quantum theory by appealing to an analogy with classical gauge theories, and specifically the example of the Maxwell theory of electromagnetism. Just as different choices of gauge for a given classical gauge theory make different calculations or properties of the theory more or less manifest—that is, each choice of gauge inevitably involves trade-offs—we see that switching from the "unitary gauge" corresponding to the original version of our interpretation of quantum theory to the "Lorenz gauge" in which it looks more like a density-matrix-centered version of the many-worlds interpretation makes the locality and Lorentz covariance of our interpretation more manifest at the cost of obscuring our interpretation's underlying ontology and the meaning of probability.

Seen from this perspective, we can also better understand why it is so challenging [69] to make sense of a many-worlds-type interpretation as an ontologically and epistemologically reasonable interpretation of quantum theory: Attempting to do so leads to as much metaphysical difficulty as trying to make sense of the Lorenz gauge of Maxwell electromagnetism as an "ontologically correct interpretation" of the Maxwell theory.[10] Hence, taking a lesson from classical gauge theories, we propose that many-worlds-type interpretations should instead be regarded as being merely a convenient mathematical tool—a particular "gauge choice"—for establishing definitively that a given "unitary-gauge" interpretation of quantum theory like our own is ultimately consistent with locality and Lorentz invariance.

[9] Observe that the eigenbasis of each system's density matrix therefore defines a preferred basis *for that system alone*—that is, in a system-centric manner. We *do not* assume the sort of universe-spanning preferred basis shared by all systems that is featured in the traditional many-worlds interpretation; such a universe-spanning preferred basis would lead to new forms of nonlocality, as we describe in our companion paper [1].

[10] Indeed, in large part for this reason, some textbooks [70] develop quantum electrodynamics fundamentally from the perspective of Weyl-Coulomb gauge $A^0 = \vec{\nabla} \cdot \vec{A} = 0$.

IV. CONCLUSION

In this letter and in a more extensive companion paper [1], we have introduced a new, conservative interpretation of quantum theory that threads a number of key requirements that we feel are insufficiently addressed by other interpretations. In particular, our interpretation identifies a definite ontology for every quantum system, as well as dynamics for that ontology based on the overall time evolution of the system's density matrix, and sews together ontologies for parent systems and their subsystems in a natural way.

Falsifiability and the Role of Decoherence

As some other interpretations do, our own interpretation puts decoherence in a central role for transforming the Born rule from an axiomatic postulate into a derived consequence as part of our larger approach to resolving the measurement problem of quantum theory. We regard it as a positive feature of our interpretation that falsification of these decoherence-based claims would mean falsification of our interpretation. We therefore take great interest in the ongoing arms race between proponents and critics of decoherence, in which critics offer up examples of decoherence coming up short[71–75] and thereby push proponents to argue that increasingly realistic measurement set-ups involving non-negligible environmental interactions resolve the claimed inconsistencies [30, 54, 76, 77].

ACKNOWLEDGMENTS

J. A. B. has benefited tremendously from personal communications with Matthew Leifer, Timothy Hollowood, Francesco Buscemi, Alisa Bokulich, Elise Crull, and Gregg Jaeger. D. K. thanks Gaurav Khanna, Pontus Ahlqvist, Adam Brown, Darya Krym, and Paul Cadden-Zimansky for many useful discussions on related topics, and has been supported in part by FQXi minigrant #FQXi-MGB-1322. Both authors have greatly appreciated their interactions with the Harvard Philosophy of Science group and its organizers Andrew Friedman and Elizabeth Petrik, as well as with Scott Aaronson, Ned Hall, and Brian Greene. Both authors are indebted to Steven Weinberg, who has generously exchanged relevant ideas and whose own recent paper [78] also explores the idea of reformulating quantum theory in terms of density matrices. In addition, the work of Pieter Vermaas has been a continuing source of inspiration for both authors, as have many conversations with Allan Blaer. The authors would also like to thank Perimeter Institute, and, in particular, Lucien Hardy and Matthew Pusey, for graciously inviting and hosting them to share their results.

This research was supported in part by Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Economic Development & Innovation.

[1] J. A. Barandes and D. Kagan, (2014), to be published, arXiv:1405.6755 [quant-ph].

[2] S. Weinberg and J. N. A. i. Matthews, Physics Today (2013), 10.1063/PT.4.2502, in particular, "... Some very good theorists seem to be happy with an interpretation of quantum mechanics in which the wavefunction only serves to allow us to calculate the results of measurements. But the measuring apparatus and the physicist are presumably also governed by quantum mechanics, so ultimately we need interpretive postulates that do not distinguish apparatus or physicists from the rest of the world, and from which the usual postulates like the Born rule can be deduced. This effort seems to lead to something like a 'many worlds' interpretation, which I find repellent. Alternatively, one can try to modify quantum mechanics so that the wavefunction does describe reality, and collapses stochastically and nonlinearly, but this seems to open up the possibility of instantaneous communication".

[3] S. Weinberg, *Lectures on Quantum Mechanics*, 1st ed. (Cambridge University Press, 2012) p. 95, in particular, the end of Section 3.7 ("The Interpretations of Quantum Mechanics"): "... [I]t would be disappointing if we had to give up the 'realist' goal of finding complete descriptions of physical systems, and of using this description to derive the Born rule, rather than just assuming it. We can live with the idea that the state of a physical system is described by a vector in Hilbert space rather than by numerical values of the positions and momenta of all the particles in the system, but it is hard to live with no description of physical states at all, only an algorithm for calculating probabilities. My own conclusion (not universally shared) is that today there is no interpretation of quantum mechanics that does not have serious flaws, and that we ought to take seriously the possibility of finding some more satisfactory other theory, to which quantum mechanics is merely a good approximation".

[4] M. Tegmark, Fortschritte der Physik **46**, 855 (1998), arXiv:quant-ph/9709032 [quant-ph].

[5] M. Schlosshauer, J. Kofler, and A. Zeilinger, Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **44**, 222 (2013), arXiv:1301.1069 [quant-ph].

[6] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Berlin: Springer, 1932).

[7] N. P. Landsman, in *"Handbook of the Philosophy of Science"* (Elsevier, 2007) pp. 417–553, with a review of the Heisenberg cut and its history in Section 3.2 ("Object and apparatus: the Heisenberg cut"), arXiv:quant-ph/0506082 [quant-ph].

[8] M. F. Pusey, J. Barrett, and T. Rudolph, Nature Physics **8**, 475 (2012), arXiv:1111.3328 [quant-ph].

[9] J. Barrett, E. G. Cavalcanti, R. Lal, and O. J. E. Maroney, (2013), arXiv:1310.8302 [quant-ph].

[10] R. Colbeck and R. Renner, (2013), arXiv:1312.7353 [quant-ph].

[11] N. Harrigan and R. W. Spekkens, Foundations of Physics **40**, 125 (2010), arXiv:0706.2661 [quant-ph].

[12] G. C. Ghirardi, A. Rimini, and T. Weber, Physical Review D **34**, 470 (1986).

[13] E. C. G. Sudarshan, P. M. Mathews, and J. Rau, Physical Review **121**, 920 (1961).

[14] T. F. Jordan and E. C. G. Sudarshan, Journal of Mathematical Physics **2**, 772 (1961).

[15] M.-D. Choi, Linear Algebra and its Applications **10**, 285 (1975).

[16] B. Schumacher and M. Westmoreland, *Quantum Processes[,] Systems, and Information*, 1st ed. (Cambridge University Press, 2010) pp. 453–462, Appendix D ("Generalized Evolution"). Note that the authors' general ensemble argument for linearity implicitly assumes that the density matrix describes a proper mixture.

[17] M. B. Ruskai, Reviews in Mathematical Physics **6**, 1147 (1994), with a proof of the inequality in Section 1.6 ("Trace Norm Relative Entropy").

[18] H.-P. Breuer, E.-M. Laine, and J. Piilo, Physical Review Letters **103**, 210401 (2009), arXiv:0908.0238 [quant-ph].

[19] E.-M. Laine, J. Piilo, and H.-P. Breuer, Physical Review A **81**, 062115 (2010), arXiv:1002.2583 [quant-ph].

[20] F. Buscemi, (2013), arXiv:1307.0363 [quant-ph].

[21] G. Lindblad, Communications in Mathematical Physics **48**, 119 (1976).

[22] W. C. Myrvold, Foundations of Physics **32**, 1773 (2002), arXiv:quant-ph/0209109 [quant-ph].

[23] H. P. Krips, Philosophy of Science **36**, 145 (1969), where we note that although van Frassen is usually credited with the idea of the modal interpretations, and Vermaas and Dieks for its spectral version, Krips appears to have arrived at these ideas first.

[24] H. P. Krips, American Journal of Physics **43**, 420 (1975).

[25] H. P. Krips, *The Metaphysics of Quantum Theory* (Clarendon Press Oxford, 1987).

[26] B. C. Van Fraassen, in *"Paradigms and Paradoxes: The Philosophical Challenge of the Quantum Domain"*, edited by R. Colodny (University of Pittsburgh Press, 1972) pp. 303–366.

[27] N. D. Cartwright, Philosophy of Science **41**, 199 (1974).

[28] P. E. Vermaas and D. Dieks, Foundations of Physics **25**, 145 (1995).

[29] G. Bacciagaluppi and W. M. Dickson, Foundations of Physics **29**, 1165 (1999).

[30] P. E. Vermaas, *A Philosopher's Understanding of Quantum Mechanics: Possibilities and Impossibilities of a Modal Interpretation* (Cambridge University Press, 1999) with a discussion of realistic measurement devices and how they evade criticisms of the modal interpretations in Chapter 10 ("The measurement problem").

[31] L. de Broglie, *An introduction to the Study of Wave Mechanics* (E. P. Dutton and Company, Inc., 1930).

[32] D. Bohm, Physical Review **85**, 166 (1952).

[33] D. Bohm, Physical Review **85**, 180 (1952).

[34] H. Everett III, Reviews of Modern Physics **29**, 454 (1957).

[35] H. Everett III, B. S. DeWitt, and N. Graham, *The many-worlds interpretation of quantum mechanics* (Princeton University Press, 1973).

[36] D. Deutsch, International Journal of Theoretical Physics **24**, 1 (1985).

[37] D. Wallace, Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **33**, 637 (2002), arXiv:quant-ph/0103092 [quant-ph].
[38] P. Pearle, Physical Review A **39**, 2277 (1989).
[39] A. Bassi and G. C. Ghirardi, Physical Reports **379**, 257 (2003), arXiv:quant-ph/0302164 [quant-ph].
[40] S. Weinberg, (2013), arXiv:1109.6462 [quant-ph].
[41] A. Aguirre and M. Tegmark, Physical Review D **84**, 105002 (2011), arXiv:1008.1066 [quant-ph].
[42] J. B. Hartle, Physical Review D **44**, 3173 (1991).
[43] M. Gell-Mann and J. B. Hartle, (2013), arXiv:1312.7454 [quant-ph].
[44] D. Deutsch, (1999), arXiv:quant-ph/9906015 [quant-ph].
[45] D. Wallace, (2002), arXiv:quant-ph/0211104 [quant-ph].
[46] C. A. Fuchs and R. Schack, (2013), arXiv:1301.3274 [quant-ph].
[47] E. Farhi, J. Goldstone, and S. Gutmann, Annals of Physics **192**, 368 (1989).
[48] R. V. Buniy, S. D. H. Hsu, and A. Zee, Physics Letters B **640**, 219 (2006), arXiv:hep-th/0606062 [hep-th].
[49] L. E. Ballentine, Reviews of Modern Physics **42**, 358 (1970).
[50] G. Bene, Physica A: Statistical Mechanics and its Applications **242**, 529 (1997), arXiv:quant-ph/9703021 [quant-ph].
[51] G. Bene and D. Dieks, Foundations of Physics **32**, 645 (2002), arXiv:quant-ph/0112134 [quant-ph].
[52] C. Rovelli, International Journal of Theoretical Physics **35**, 1637 (1996), arXiv:quant-ph/9609002 [quant-ph].
[53] J. Berkovitz and M. Hemmo, Foundations of Physics **35**, 373 (2005).
[54] T. J. Hollowood, (2013), arXiv:1312.3427 [quant-ph].
[55] T. J. Hollowood, (2013), arXiv:arXiv:1312.4751 [quant-ph].
[56] D. Z. Albert and B. Loewer, Synthese **77**, 195 (1988).
[57] W. M. Dickson, Foundations of Physics Letters **8**, 401 (1995).
[58] G. Bacciagaluppi and W. M. Dickson, (1997), arXiv:quant-ph/9711048 [quant-ph].
[59] J. Earman and L. Ruetsche, "Philosophy of Science" **72**, 557 (2005).
[60] G. 't Hooft, in *Quo Vadis Quantum Mechanics?*, The Frontiers Collection (Springer, 2005) pp. 99–111, arXiv:quant-ph/0212095 [quant-ph].
[61] G. 't Hooft, (2014), arXiv:1405.1548 [quant-ph].
[62] S. L. Adler, (2002), arXiv:hep-th/0206120 [hep-th].
[63] A. D. Linde, Modern Physics Letters A **1**, 81 (1986).
[64] J. Garriga and A. Vilenkin, Physical Review D **57**, 2230 (1998), arXiv:astro-ph/9707292 [astro-ph].
[65] A. H. Guth, Journal of Physics A: Mathematical and Theoretical **40**, 6811 (2007), arXiv:hep-th/0702178 [hep-th].
[66] W. M. Dickson and R. Clifton, in *"The Modal Interpretation of Quantum Mechanics"* (Springer, 1998) pp. 9–47.
[67] L. Hardy, Physical Review Letters **68**, 2981 (1992).
[68] W. C. Myrvold, The British Journal for the Philosophy of Science **60**, 635 (2009).
[69] S. D. H. Hsu, Modern Physics Letters A **27** (2012), arXiv:1110.0549 [quant-ph].
[70] S. Weinberg, *The Quantum Theory of Fields*, Vol. 1 (Cambridge University Press, 1996) for a development of classical gauge theories from the perspective of Weyl-Coulomb gauge, in which the gauge potential is not a proper Lorentz four-vector, see Section 5.9.
[71] D. Z. Albert and B. Loewer, PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association **1**, 277 (1990).
[72] D. Z. Albert and B. Loewer, Synthese **86**, 87 (1991).
[73] D. Z. Albert and B. Loewer, Foundations of Physics Letters **6**, 297 (1993).
[74] D. Z. Albert, *Quantum Mechanics and Experience*, 1st ed. (Harvard University Press, 1994) for the question of degeneracies, see Appendix: The Kochen-Healy-Dieks Interpretations.
[75] D. N. Page, (2011), arXiv:1108.2709 [hep-th].
[76] G. Bacciagaluppi and M. Hemmo, Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **27**, 239 (1996).
[77] T. J. Hollowood, (2013), arXiv:1302.4228 [quant-ph].
[78] S. Weinberg, (2014), arXiv:1405.3483 [quant-ph].